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DIFFUSION—CONVECTIVE VAPORIZATION OF DROPS BY INTENSIVE OPTICAL RADIATION WITH AN ALLOWANCE FOR THE TEMPERATURE DEPENDENCES OF THE TRANSFER COEFFICIENTS

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A quasistationary solution is obtained for diffusion—convective vaporization of drops by intensive optical radiation with an allowance for the temperature dependences of the transfer coefficients. Comparisons with experimental data are provided.

Convective drop vaporization in a gaseous medium by intensive optical radiation was considered in [1-5]. In [1, 2] it was assumed that the temperature drop values in the gas surrounding a drop are small during the vaporization process, while the transfer coefficients $\varkappa$ and D were assumed to be constant (independent of the temperature). Attempts were made in [3, 4] to account for the temperature dependence of the transfer coefficients in convective drop vaporization. However, the actual temperature dependences of $\varkappa$ and D [6] are substantially different from those used in [3, 4]. Consideration of the temperature dependences of $\varkappa$ and D is especially important in the case of arbitrary drops in temperature, since the accuracy in describing the heat and mass transport in the gas surrounding a drop determines

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the dynamics of drop vaporization; this problem is the subject of the present paper. We have compared some of our results with experimental data [7].

Consider a spherical drop in a binary gaseous medium consisting of vapor molecules of the drop substance and molecules of an inert gas, which does not condense at the drop surface. Assume that optical radiation with the wavelength λ and the energy flux density $I_0(t)$ is incident to a drop whose initial radius is r_∞ and whose temperature is T_∞ (equal to the ambient temperature) following the instant of time $t = 0$. The drop absorbs the optical radiation energy and is heated, which causes heat exchange between the drop and the ambient as a result of thermal conductivity and evaporation. For drops with $r_0 < \lambda$, the density of energy release q due to radiation energy absorption is virtually uniform [5] and is determined by the expression

$$q = \frac{3I_0K_a(r_0, \lambda)}{4r_0}. \tag{1}$$

The system of equations describing the heating and vaporization of the drop has the following form in the uniform-temperature approximation throughout its volume [8, 9]:

$$\begin{gathered}\rho_{su}c_{su}V_0\frac{dT_0}{dt} = qV_0 - \bar{j}_\varepsilon S_0,\\ \rho_{su}\frac{dV_0}{dt} = -\bar{j}S_0,\end{gathered} \tag{2}$$

while the initial conditions are

$$r_0(t=0) = r_\infty, \quad T_0(t=0) = T_\infty. \tag{3}$$

Consideration of drop vaporization in the $T_\infty \leqq T_0 < T_c$ temperature range, when for $T_0 \lesssim T_c$ the saturated vapor pressure of the drop substance is comparable to the gaseous ambient pressure, entails consideration of convective (hydrodynamic) heat and mass transfer in the medium. The drop evaporates with spherical symmetry for radial convection in the gaseous medium. We consider the processes of heat and mass transport in the quasistationary approximation, since the characteristic transient periods of the thermal conductivity, diffusion, and convection processes are much shorter than the characteristic times of drop heating and vaporization [9]. Moreover, since the mass velocities in convection are much lower than the velocity of sound in gases, the gas pressure is uniform and constant throughout the volume. The system of equations in a spherical coordinate system describing quasistationary convective heat and mass transport in the ambient gas during drop vaporization has the following form if we neglect thermal diffusion and heat transport due to diffusion [3]:

$$p = \rho RT = p_\infty = \text{const}, \tag{4}$$

$$\rho v\frac{dc_1}{dr} = \frac{1}{r^2}\frac{d}{dr}\left(r^2\rho D\frac{dc_1}{dr}\right), \tag{5}$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho v) = 0, \tag{6}$$

$$\rho c_v v\frac{dT}{dr} = \frac{1}{r^2}\frac{d}{dr}\left(r^2\varkappa\frac{dT}{dr}\right) - \frac{p_\infty}{r^2}\frac{d}{dr}(r^2 v), \tag{7}$$

while the boundary conditions are

$$\begin{gathered}r = r_0, \quad \rho = \bar{\rho}, \quad v = \bar{v}, \quad T = \bar{T}, \quad c_1 = \bar{c}_1,\\ \left.\left(\rho_2 v - \rho D\frac{dc_2}{dr}\right)\right|_{r_0} = 0,\end{gathered} \tag{8}$$

$$r \to \infty, \quad \rho \to \rho_\infty, \quad v \to 0, \quad T \to T_\infty, \quad c_1 \to c_{1\infty}, \tag{9}$$

where $\rho = \rho_1 + \rho_2$, $c_1 = \rho_1/\rho$, and, generally, $R = R(R_1, R_2, c_1)$, $c_V = c_V(c_{V1}, c_{V2}, c_1)$, $\varkappa = \varkappa(T, c_1)$; $D = D(T)$. Equation (4) expresses the condition of constant pressure in the medium, expression (6) constitutes the equation of continuity, and (5) and (7) are equations of vapor diffusion and of heat transport with an allowance for convection. By integrating (6) and (5) and using (8), we obtain

$$\frac{r^2 p_\infty}{RT} D \frac{dc_1}{dr} = r_0^2 \bar{j} (c_1 - 1), \tag{10}$$

where $\bar{j} = \bar{\rho}\bar{v}$ is the density of the vapor mass flux from the drop with an allowance for (8).

Assuming, furthermore, that R and c_V are constant quantities, R = const, c_V = const, we obtain the following from (7) by taking into account (6):

$$r^2 \varkappa \frac{dT}{dr} = r_0^2 \bar{j} c_p (T - B), \tag{11}$$

where $c_p = c_V + R$; the integration constant B is determined from (8) and (9). If $\varkappa$ depends only on T, $\varkappa(T)$, we obtain the following from (7) by considering (4) and (6):

$$\int \frac{\varkappa(T)\, dT}{T - B} = -\frac{r_0^2 \bar{j} c_p}{r} + C, \tag{12}$$

where the integration constant C is determined from (8) and (9). Equation (12) describes the temperature distribution T(r) for an arbitrary $\varkappa(T)$ relationship. In particular, the thermal conductivity coefficient $\varkappa$ and the diffusion coefficient D can be represented with sufficiently high accuracy in a wide temperature range in the form of power functions of the temperature [6]:

$$\varkappa = \varkappa_\infty \left(\frac{T}{T_\infty}\right)^{p_1}; \quad D = D_\infty \left(\frac{T}{T_\infty}\right)^{b}, \tag{13}$$

where $\varkappa_\infty = \varkappa(T_\infty)$, $D_\infty = D(T_\infty)$, $p_1 \geqq 0$, $b \geqq 0$ for gaseous media. For $\varkappa(T)$ defined by (13) and the $0 \leqq p_1 \leqq 1$ range, which is of the greatest interest for practical purposes, we obtain the following solutions from (12) with an allowance for (8) and (9):

$$\begin{gathered}
p_1 = 0, \ T = T_\infty + \frac{\bar{T} - T_\infty}{1 - \exp\left(-\dfrac{r_0 \bar{j} c_p}{\varkappa_\infty}\right)} \left[1 - \exp\left(-\frac{r_0^2 \bar{j} c_p}{\varkappa_\infty r}\right)\right]; \\
p_1 = \frac{1}{2}, \ \ T^{1/2} - T_\infty^{1/2} + \frac{B^{1/2}}{2} \ln \frac{(T^{1/2} - B^{1/2})(T_\infty^{1/2} + B^{1/2})}{(T^{1/2} + B^{1/2})(T_\infty^{1/2} - B^{1/2})} = \\
= -\frac{T_\infty^{1/2} r_0^2 \bar{j} c_p}{2\varkappa_\infty r}; \quad p_1 = \frac{2}{3}, \ \ \frac{3}{2}(T^{2/3} - T_\infty^{2/3}) + \\
+ \sqrt{3} B^{2/3} \left(\operatorname{arctg} \frac{2T^{1/3} + B^{1/3}}{\sqrt{3} B^{1/3}} - \operatorname{arctg} \frac{2T_\infty^{1/3} + B^{1/3})}{\sqrt{3} B^{1/3}}\right) - \\
- \frac{B^{2/3}}{2} \ln \frac{(B^{2/3} + B^{1/3} T^{1/3} + T^{2/3})(B^{1/3} - T_\infty^{1/3})^2}{(B^{1/3} - T^{1/3})^2 (B^{2/3} + B^{1/3} T_\infty^{1/3} + T_\infty^{2/3})} = -\frac{T_\infty^{2/3} r_0^2 \bar{j} c_p}{\varkappa_\infty r}; \\
p_1 = \frac{3}{4}, \ \ \frac{4}{3}(T^{3/4} - T_\infty^{3/4}) + 2B^{3/4}\left(\operatorname{arctg} \frac{T^{1/4}}{B^{1/4}} - \operatorname{arctg} \frac{T_\infty^{1/4}}{B^{1/4}}\right) + \\
+ B^{3/4} \ln \frac{(B^{1/4} + T_\infty^{1/4})(B^{1/4} - T^{1/4})}{(B^{1/4} - T_\infty^{1/4})(B^{1/4} + T^{1/4})} = -\frac{T_\infty^{3/4} r_0^2 \bar{j} c_p}{\varkappa_\infty r}; \\
p_1 = 1, \ T - T_\infty + B \ln \frac{T - B}{T_\infty - B} = -\frac{T_\infty r_0^2 \bar{j} c_p}{\varkappa_\infty r}.
\end{gathered} \tag{14}$$

A particular solution of the temperature distribution in convective vaporization was obtained in [2] for $p_1 = 0$ and in [4] for $p_1 = 1/2$. For $r = r_0$ expressions for $\bar{j}$ can be obtained from (14), for instance

$$\begin{gathered}
p_1 = \frac{3}{4}; \ \bar{j} = \frac{\varkappa_\infty}{c_p \bar{T}_\infty^{3/4} r_0} \left[B^{3/4} \ln \frac{(B^{1/4} + \bar{T}^{1/4})(B^{1/4} - T_\infty^{1/4})}{(B^{1/4} - \bar{T}^{1/4})(B^{1/4} + T_\infty^{1/4})} - \right. \\
\left. - \frac{4}{3}(\bar{T}^{3/4} - T_\infty^{3/4}) - 2B^{3/4}\left(\operatorname{arctg} \frac{\bar{T}^{1/4}}{B^{1/4}} - \operatorname{arctg} \frac{T_\infty^{1/4}}{B^{1/4}}\right)\right].
\end{gathered} \tag{15}$$

The nonlinear quasistationary temperature distribution around an evaporating drop in the absence of convection (v = 0) was obtained in [9] with an allowance for (13):

$$T = T_\infty \left[1 + \frac{r_0}{r}\left\{\left(\frac{\bar{T}}{T_\infty}\right)^{p_1+1} - 1\right\}\right]^{\frac{1}{p_1+1}}. \tag{16}$$

Expansion of (14) with respect to the small parameters $r_0\bar{j}c_p/\varkappa_\infty$ (for $p_1 = 0$) and with respect to T/B and T_∞/B (for $1/2 \leq p_1 \leq 1$) leads to (16).

Expressions for $\bar{j}$ from an evaporating drop were obtained in [10] by considering (13) in the absence of convection (v = 0) in the case of diffusion removal of vapor ($\bar{c}_1$, $c_{1\infty} \ll 1$), arbitrary temperature drops, and p = const. Expansion of the $\bar{j}$ values obtained from (14), for instance (15), with respect to the small parameters $\bar{c}_1$ and $c_{1\infty}$, leads to the expressions given in [10]. Thus, the expressions for T and $\bar{j}$ accounting for convection admit of limiting passage to expressions for diffusion vaporization. If we know T(r) (14), we can determine $\rho(r)$ from (4) and then obtain the distribution v(r) from the expression for $\bar{j}$ [for instance, (15)].

Considering (13), we integrate Eq. (10) in the following form:

$$\ln\frac{c_{1\infty} - 1}{c_1 - 1} = \frac{r_0^2\bar{j}RT_\infty^b}{p_\infty D_\infty}\int_r^\infty \frac{T(r)^{1-b}dr}{r^2}; \tag{17}$$

After the substitution of T(r), the above expression yields the distribution $c_1(r)$. For instance, for b = 1, we obtain from (17)

$$c_1 = 1 + (c_{1\infty} - 1)\exp\left(\frac{r_0}{r}\ln\frac{\bar{c}_1 - 1}{c_{1\infty} - 1}\right).$$

Combining (10) and (11), we arrive at the following equation:

$$\ln\frac{c_1 - 1}{\bar{c}_1 - 1} = \frac{R\varkappa_\infty T_\infty^{b-p_1}}{p_\infty c_p D_\infty}\int_{\bar{T}}^{T(r)} \frac{T^{p_1-b+1}}{T - B}\,dT, \tag{18}$$

which relates c_1 to T. For instance, for $p_1 = b = 0$, Eq. (18) has the following solution:

$$\ln\frac{c_1 - 1}{\bar{c}_1 - 1} = \frac{R\varkappa_\infty}{p_\infty c_p D_\infty}\left(T - \bar{T} + B\ln\frac{T - B}{\bar{T} - B}\right).$$

One of the most important cases of practical significance is that of $p_1 = b - 1$, since the exponents p_1 and b usually lie within the ranges $1/2 < p_1 < 1$, $3/2 < b < 2$ [6]. We obtain from (18)

$$c_1 = 1 + (\bar{c}_1 - 1)\left(\frac{T - B}{\bar{T} - B}\right)^\alpha, \tag{19}$$

where $\alpha = R\varkappa_\infty T_\infty/(p_\infty c_p D_\infty)$. Using (8) and (9), we determine B by means of Eq. (19):

$$B = \frac{\bar{T}\left(\frac{c_{1\infty} - 1}{\bar{c}_1 - 1}\right)^{1/\alpha} - T_\infty}{\left(\frac{c_{1\infty} - 1}{\bar{c}_1 - 1}\right)^{1/\alpha} - 1}. \tag{20}$$

Consider the vaporization of water drops in air at atmospheric pressure under diffusion-convective conditions under the action of continuous optical radiation with $\lambda = 10.6$ μm. We assume that the evaporation (condensation) coefficient of water is equal to unity, and we use the expression for $K_a(r_0)$ at $\lambda = 10.6$ μm, borrowed from [8]. In correspondence with [6], the parameters D_∞ and b in the temperature dependence of the coefficient of water vapor diffusion in air (13) are assumed to be $D_\infty = 2.16 \cdot 10^{-5}$ m²/sec and b = 1.75. For these values of D_∞ and b, (13) involves a mean error of not more than 5% in the 273 < T < 493°K range. The temperature jump at the boundary of an evaporating drop with r_∞ = 5-40 μm does not exceed

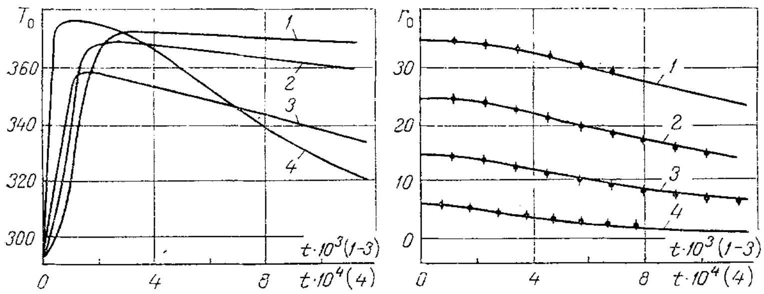

Fig. 1. Radius r_0 and temperature T_0 of a drop as functions of time t for r = 15 (1), 25 (2), and 35 μm (3) and $I_0 = 9.3\cdot 10^3$ kW/m²; r_∞ = 6 μm (4) and $I_0 = 1.3\cdot 10^5$ kW/m². Points represent experimental data [7]; solid curves provide theoretical results; vertical segments represent experimental errors; r_0, μm; T_0, K; t, sec.

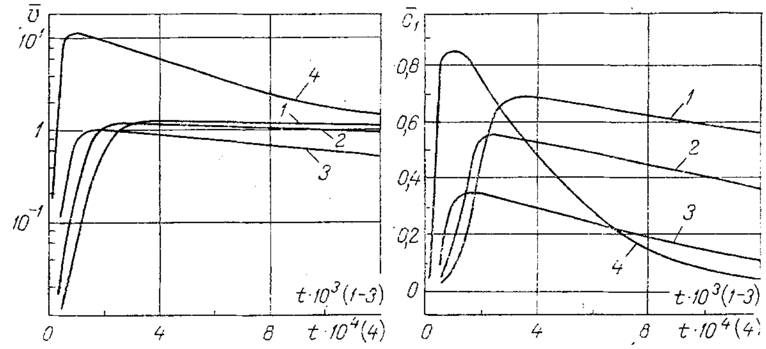

Fig. 2. Velocity $\bar{v}$ and concentration $\bar{c}_1$ at the drop surface as functions of time t for r_∞ = 15 (1), 25 (2), and 35 μm (3) and $I_0 = 9.3\cdot 10^3$ kW/m²; r_∞ = 6 μm (4) and $I_0 = 1.3\cdot 10^5$ kW/m²; $\bar{v}$, m/sec; t, sec.

1-3°K [8, 9]. Consequently, for the sake of simplicity, we neglect the temperature jump at the drop boundary and assume that $\bar{T} = T_0$. The vapor density jump at the drop boundary is accounted for by kinetic analysis of the evaporation and condensation processes, similar to what was done in [8]. The temperature dependence $\varkappa(T)$ [6] is approximated by means of (13) for $\varkappa_\infty = 2.4\cdot 10^{-2}$ W/(m·deg) and $p_1 = 0.75$ in the 273 < T < 473°K range. In our case, p_1 = 0.75, b = 1.75, and $p_1 = b - 1$. Consequently, we use (15) to determine $\bar{j}$ and (20) to determine B. The relationships $L(T_0)$ and $n_0(T_0)$ were determined in accordance with [6]. The energy flux density $\bar{j}_\varepsilon$ with regard to determination of the flux density of the heat $\bar{j}_h$ removed through the thermal conductivity mechanism, $\bar{j}_h = [-\varkappa(dT/dr)]|_{r_0}$ [see (11)], is determined by the relationship

$$\bar{j}_\varepsilon = \bar{j}\,[c_p(B - \bar{T}) + L]. \tag{21}$$

We performed numerical calculations of the system of equations (1), (2), (15), (20), and (21) for the initial conditions (3) for water drops with r_∞ = 15, 25, 35 μm at $I_0 = 9.3\cdot 10^3$ kW/m² and with r_∞ = 6 μm at $I_0 = 1.3\cdot 10^5$ kW/m² for comparison with experimental data [7]. Figure 1 shows the experimental and theoretical relationships $r_0(t)$ and $T_0(t)$. The theoretical relationships $r_0(t)$ are in good agreement with experimental data. It should be mentioned that, in the process of heating and evaporation, the maximum temperature of a drop with r_∞ = 6 μm is somewhat higher than the T_b value (by ~ 4°K). However, consideration of the vapor density jump at the surface of an evaporating drop brings about a situation where the vapor pressure does not exceed the atmospheric pressure, so that use of the diffusion–convective conditions of drop evaporation for p = const is admissible. After the maximum temperature is reached during the process of drop vaporization, T_0 decreases. Figure 2 shows the theoretical dependences of the vapor concentration $\bar{c}_1$ and the velocity $\bar{v}$ directly at the drop surface on the time t for the same calculation variants. The increase in the drop temperature at the initial vaporization stage at $0 < t < t_m$ (t_m is the time in which the drop temperature reaches its maximum value) causes displacement of the air in the vicinity of the drop, while the decrease in the drop temperature during the evaporation process at $t > t_m$ results in a gradual return of air to the drop.

The characteristic values of the convective motion velocity $\bar{v}$ at the drop surface lie in the ~1-10 m/sec range and are determined by the variation in the drop temperature and radius in time during the vaporization process.

NOTATION

$\varkappa$, thermal conductivity coefficient of the gaseous medium; D, diffusion coefficient of the drop substance vapor in the ambient; L, evaporation heat per unit mass of the drop substance; r_0, present drop radius; K_a, efficiency factor of radiation absorption by the drop; ρ_{su}, density of the drop substance; c_{su}, specific heat of the drop substance; $V_0 = {}^4/_3\pi r_0{}^3$, drop volume; T_0, temperature of the drop; $\bar{j}_\varepsilon$, energy flux density at the drop surface; $S_0 = 4\pi r_0{}^2$, surface area of the drop; $\bar{j}$, mass flux density at the drop surface; T_b, boiling point of the drop substance; p, pressure; ρ, density; T, temperature; R, gas constant; v, velocity; r, radial coordinate; ρ_i, density of the i-th component; c_v, specific heat at constant volume; c_i, concentration of the i-th component; c_p, specific heat at constant pressure; n_0, density of saturated vapor of the drop substance at T_0; R_i, gas constant of the i-th component; c_{vi}, specific heat at constant volume of the i-th component. The subscripts denote the following: 1, vapor of the drop substance; 2, gas; 0, drop parameters; ∞, initial parameters.

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